Dynamics of Two-Dimensional Dark Solitons in a Smoothly Inhomogeneous Bose-Einstein Condensate

L.A. Smirnov*, ***V.A. Mironov***
Institute of Applied Physics RAS, Nizhny Novgorod, Russia
**e-mail address: smirnov_lev@appl.sci-nnov.ru*

We developed an asymptotic theory describing the dynamics of two-dimensional dark quasi-solitons in a smoothly inhomogeneous two-dimensional Bose-Einstein condensate (BEC) with repulsive interaction between the atoms. In particular, this is the problem to which the analysis of the vortex pair behaviour in the BEC in a disk-shaped trap is reduced [1,2].

The BEC dynamics is successfully described in the mean field approximation using the wavefunction satisfying the so called Gross-Pitaevskii (GP) equation. In a homogeneous BEC, the GP equation has a solution in the form of two-dimensional dark solitons moving with constant subsonic velocity υ. Depending on υ, two-dimensional dark solitons can be “vortex” and “vortex-free.” In the first case, they represent the so-called vortex pairs (or dipoles), in which the BEC concentration at two points (topological defects) located on the line perpendicular to the direction of motion become zero. Vortex-free solitons are characterized by the absence of zeros of the concentration. In the weakly linear limit, where the velocity tends to the sound velocity ($\upsilon \to 1$), they coincide with known solutions of the Kadomtsev-Petviashvili (KP) equation. The properties of two-dimensional solitons were studied in the papers [3-5]. In addition this works, we found an approximation formula for the dependence of the momentum on the energy of two-dimensional dark solitons.

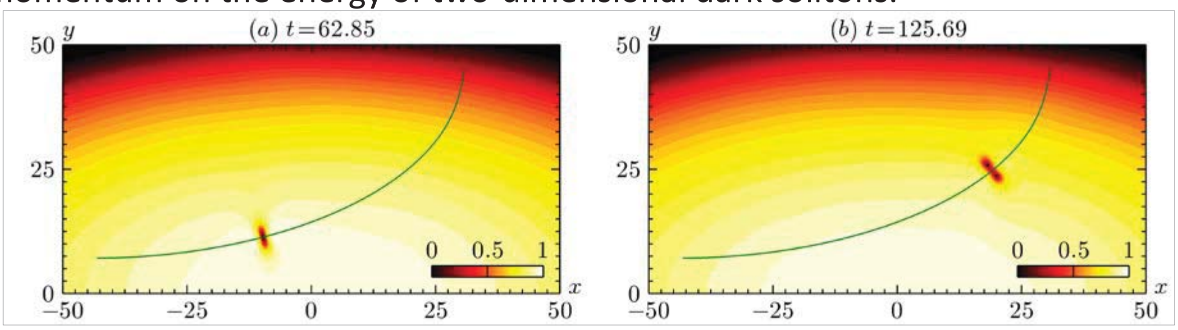

Fig.1. The comparison of the results of analytical studies (solid line) with direct numerical simulation of the GP equation.

We have studied the behaviours of localized two-dimensional structures (quasi-solitons) in a smoothly inhomogeneous BEC with repulsive interaction between the atoms. It is established that the dynamics of such structures is determined by the distribution of the undisturbed condensate density and also by their energy.

We derived of a self-consistent system of ordinary differential equations relating the integral characteristics of the quasi-soliton and the trajectory of its propagation in a smoothly inhomogeneous BEC. This system is reduced to the usual geometric-optical form with the only difference that the effective refractive index is determined not only by the properties of the undisturbed condensate, but also by the energy of the quasi-soliton formation. Using these equations, we examined specific features of the propagation of two-dimensional quasi-solitons with different initial energy in a non-uniform condensate.

By comparing with the results of numerical calculations performed directly within the GP equation, it is demonstrated that the theory we developed adequately describes the dynamics of two-dimensional dark quasi-solitons in a smoothly inhomogeneous BEC (Fig.1).

It is also shown, that during penetration into the denser condensate, the vortex pairs change their structure and convert into vortex-free quasi-solitons (close in structure to the two-dimensional solitons of the KP equation), and vice versa, the vortex-free solitons, as they enter the region of the looser condensate, convert into vortex dipoles (vortex-antivortex pairs) (Fig.1).

References

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