

3D Euler Equations and Ideal MHD Mapped to Regular Systems: Probing the Finite-Time Blowup Hypothesis

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One of the most important unsolved problems in Mathematics entails a simple question: *are solutions to the three-dimensional Euler equations globally regular or do they blow up in a finite time?* The analogous question for Navier-Stokes equations, also unsolved, corresponds to one of the famous Millennium Prize problems [1].

While several conclusive results are available for Euler and Navier-Stokes in dimension two, and dimension three with either axial or helical symmetry, attempts to understand the regularity of three-dimensional solutions have only reached local and/or conditional analytical results.

We prove by an explicit construction that solutions to incompressible 3D Euler equations defined in the periodic cube can be mapped bijectively to a new system of equations whose solutions are globally regular.

We establish that the usual Beale-Kato-Majda criterion [2] for finite-time singularity (or blowup) of a solution to the 3D Euler system is equivalent to a condition on the corresponding *regular* solution of the new system. In the hypothetical case of Euler finite-time singularity, we provide an explicit formula for the blowup time in terms of the regular solution of the new system.

The new system is amenable to being integrated numerically using similar methods as in Euler equations. We propose a method to simulate numerically the new regular system and describe how to use this to draw robust and reliable conclusions on the finite-time singularity problem of Euler equations, based on the conservation of quantities directly related to energy and circulation.

The method of mapping to a regular system can be extended to any fluid equation that admits a Beale-Kato-Majda type of theorem, e.g. 3D Navier-Stokes, 2D and 3D magnetohydrodynamics, and 1D inviscid Burgers. We discuss briefly the case of 2D ideal magnetohydrodynamics. See reference [3] for details.

In order to illustrate the usefulness of the mapping, we provide a thorough comparison of the analytical solution versus the numerical solution in low-dimensional models that admit this mapping method: 1D inviscid Burgers equation and a new 2D vortex collapse model. We show that the numerical

integration of the mapped regular system provides results with better accuracy than the numerical integration of the original system, using the same numerical scheme, and same computation time and memory.

References

- [1] <http://www.claymath.org>
- [2] J.T. Beale et al., "Remarks on the breakdown of smooth solutions for the 3-D Euler equations", *Commun. Math. Phys.* **94**, 61 (1984).
- [3] M.D. Bustamante, "3D Euler equations and ideal MHD mapped to regular systems: Probing the finite-time blowup hypothesis", *Physica D* **240**, 1092 (2011).